

# Electromagnetic Couplings of Nucleon Resonances<sup>1,2</sup>

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## Abstract

An effective Lagrangian calculation of pion photoproduction including all nucleon resonances up to  $\sqrt{s} = 1.7$  GeV is presented. We compare our results to recent calculations and show the influence of different width parametrizations and offshell cutoffs on the photoproduction multipoles. We determine the electromagnetic couplings of the resonances from a new fit to the multipole data.

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# 1 Introduction

In recent years effective Lagrangian models have been used to calculate pion and eta photoproduction on the nucleon [1, 2, 3, 4, 5, 6].

For pion production these models were mainly applied to energies below the second resonance region, including nucleon Born terms, vector mesons and the  $\Delta(1232)$  resonance. It was found that proper unitarization is necessary to describe the pion photoproduction amplitudes in all channels with  $l \leq 1$ . Unitarity was guaranteed by explicit inclusion of  $\pi N$  rescattering [1] or by using Watson's theorem in one way or another [2].

In the case of eta production the electromagnetic couplings of the  $N(1535)$  resonance were extracted from a unitary calculation of the  $E_{0+}^p$  pion multipole and the total  $\gamma N \rightarrow \eta N$  cross section. These calculations include only the Born terms, vector mesons and the  $S_{11}$  resonances. Offshell s- and u-channel  $\Delta(1232)$  and  $N(1520)$  contributions to the pion production are neglected [5] since only the  $S_{11}$   $\pi N$  scattering channel was calculated to fit the hadronic properties of the resonances.

Unfortunately there is no "combined" model that extends the unitary calculations done in the  $\Delta(1232)$  region up to the second or even third resonance region ( $\sqrt{s} \leq 1.7$  GeV) including all resonances and calculating all multipoles.

As a first step Garcilazo et al. [3] neglected all rescattering effects in the pion photoproduction and still found a reasonable agreement in most multipole channels with  $l \leq 2$ . In the  $\Delta(1232)$  region the remaining discrepancies in the  $E_{1+}^{3/2}$  channel can be fully explained by the rescattering. But in this approach width parametrizations and additional cutoffs introduce unwanted ambiguities. Furthermore the role of the offshell contributions of the spin- $\frac{3}{2}$  resonances was not investigated systematically.

Since the pion photoproduction is the main reaction to extract the electromagnetic couplings of the nucleon resonances [7] a careful examination of the influence of all degrees of freedom present in an effective Lagrangian approach is necessary.

The aim of this paper is therefore to show how these additional degrees of freedom influence the extracted electromagnetic couplings of the nucleon resonances given by [3]. For a set of possible combinations of width parametrizations and cutoffs we fit the photoproduction multipole data either with fixed or varying spin- $\frac{3}{2}$  offshell contributions. It will be demonstrated that the extracted couplings depend heavily on these offshell contributions and that this problem can only be resolved in a complete calculation. Unfortunately the offshell contributions are most important in non-resonant multipole channels for energies away from the resonance position so that it is probably not possible to study the influence of the spin- $\frac{3}{2}$  resonances separately. Since a full calculation has only been carried out in the  $\Delta(1232)$  energy region [8] our calculation can be viewed as the starting point for further investigations. Especially an estimate of the size of different effects can be made.

As an example for this we show the contribution of the  $N(1520)$  to the  $E_{0+}^p$  pion photoproduction multipole and compare it to the known rescattering effects. It can be seen that both effects are of the same order of magnitude and that therefore the determination of the  $N(1535)$  couplings via the  $E_{0+}^p$  multipole is influenced by the presence of the  $N(1520)$ .

## 2 Model Lagrangians for pion photoproduction

Starting points are the interaction lagrangians for the hadronic and electromagnetic coupling of the contributing particles. In the following  $m, M, M_R$  are the pion, nucleon and nucleon resonance mass, respectively.

For the Born terms and the spin- $\frac{1}{2}$  resonances we use pseudovector (PV)  $\pi NN$  and pseudoscalar (PS)  $\eta NN$  coupling. For the vector mesons ( $\rho$  and  $\omega$ ) we use the Lagrangians given in [2].

Spin- $\frac{1}{2}$  ( $N(1440)$ ,  $N(1535)$ ,  $\Delta(1620)$  and  $N(1650)$ ) resonances [6]:

$$\mathcal{L}_{R_{1/2}N\pi}^{PS} = -g_\pi \bar{R} i \Gamma \vec{\tau} \vec{\pi} N + h.c., \quad (1)$$

$$\mathcal{L}_{R_{1/2}N\pi}^{PV} = -\frac{g_\pi}{M_R \pm M} \bar{R} \Gamma_\mu \vec{\tau} (\partial^\mu \vec{\pi}) N + h.c., \quad (2)$$

$$\mathcal{L}_{R_{1/2}N\gamma} = e \bar{R} (g_1^s + g_1^v \tau_3) \frac{\Gamma_{\mu\nu}}{4M} N F^{\mu\nu} + h.c., \quad (3)$$

where  $R$  is the resonance spinor. The upper sign in (2) holds for even parity, the lower sign for odd parity resonances. The operators  $\Gamma$ ,  $\Gamma_\mu$  and  $\Gamma_{\mu\nu}$  are given by

$$\Gamma = 1, \quad \Gamma_\mu = \gamma_\mu, \quad \Gamma_{\mu\nu} = \gamma_5 \sigma_{\mu\nu}, \quad (4)$$

$$\Gamma = \gamma_5, \quad \Gamma_\mu = \gamma_5 \gamma_\mu, \quad \Gamma_{\mu\nu} = \sigma_{\mu\nu}, \quad (5)$$

where (4) and (5) correspond to resonances of odd and even parities, respectively. The magnetic couplings for proton and neutron targets are  $g_1^p = g_1^s + g_1^v$  and  $g_1^n = g_1^s - g_1^v$ .  $F^{\mu\nu}$  represents the electromagnetic field tensor.

Spin- $\frac{3}{2}$  ( $\Delta(1232)$ ,  $N(1520)$  and  $\Delta(1700)$ ) resonances:

$$\mathcal{L}_{R_{3/2}N\pi} = \frac{f_\pi}{m} \bar{R}^\alpha \Theta_{\alpha\mu}(z_\pi) \left[ \begin{array}{c} 1 \\ \gamma_5 \end{array} \right] \vec{T}(\partial^\mu \vec{\pi}) N + h.c., \quad (6)$$

$$\mathcal{L}_{R_{3/2}N\gamma} = \frac{ie g_1}{2M} \bar{R}^\alpha \Theta_{\alpha\mu}(z_1) \gamma_\nu \left[ \begin{array}{c} \gamma_5 \\ 1 \end{array} \right] T_3 N F^{\nu\mu} + \quad (7)$$

$$- \frac{e g_2}{4M^2} \bar{R}^\alpha \Theta_{\alpha\mu}(z_2) \left[ \begin{array}{c} \gamma_5 \\ 1 \end{array} \right] T_3 (\partial_\nu N) F^{\nu\mu} + h.c., \quad (8)$$

$$\Theta_{\alpha\mu}(z) = g_{\alpha\mu} - \frac{1}{2}(1 + 2z)\gamma_\alpha \gamma_\mu. \quad (9)$$

For the  $N(1520)$  the proton and neutron couplings  $g_1^{p,n}$  and  $g_2^{p,n}$  are used. The operator  $\Theta_{\alpha\mu}(z)$  describes the offshell admixture of spin- $\frac{1}{2}$  fields. Some attempts have been made to fix the parameters  $z$  by examining the Rarita-Schwinger equations and the transformation properties of the interaction Lagrangians [9, 10]. Since most of the arguments presented there do not hold for composite particles and not all problems of interacting spin- $\frac{3}{2}$  fields could be solved, we treat these parameters as free and try to determine them by fitting the pion photoproduction multipoles. For a more detailed discussion see [2].

The couplings (3) - (9) can easily be compared to other choices [1, 3, 6]. The  $RN\gamma$  Lagrangians differ only by normalizations factors like  $(M_R + M)/(2M)$  or by use of the "Sachs" type couplings in the  $R_{3/2}N\gamma$  vertices.

The Lagrangian  $\mathcal{L}_{NNV}$  for the vector meson coupling is choosen in analogy to the  $NN\gamma$  case [2]:

$$\mathcal{L}_{NN\gamma} = -e\bar{N} \left\{ \frac{(1 + \tau_3)}{2} \gamma_\mu A^\mu - (\kappa^s + \kappa^v \tau_3) \frac{\sigma_{\mu\nu}}{4M} F^{\mu\nu} \right\} N, \quad (10)$$

$$\mathcal{L}_{NNV} = -g_{NNV} \bar{N} \left\{ \gamma_\mu V^\mu - K_V \frac{\sigma_{\mu\nu}}{2M} V^{\mu\nu} \right\} N, \quad (11)$$

where  $V_\mu$  is the  $\rho$  or  $\omega$  field and  $V_{\mu\nu} = \partial_\nu V_\mu - \partial_\mu V_\nu$  is the the vector meson field tensor. Otherwise it is not possible to use the VMD values for  $K_V$  which are derived from the anomalous magnetic moments  $(\kappa^s + \kappa^v \tau_3)$  of the nucleon. For means of comparison we use the same vector meson couplings as in [1, 3]:

$$\begin{aligned} g_{\omega\pi\gamma} &= 0.313, & g_{\rho^0\pi\gamma} &= 0.131, & g_{\rho^\pm\pi\gamma} &= 0.103, \\ g_{NN\omega} &= 3g_{NN\rho} = 7.98, & K_\omega &= -0.12, & K_\rho &= 3.71. \end{aligned} \quad (12)$$

For completeness we also give the electromagnetic vertex of the vector mesons [2]:

$$\mathcal{L}_{V\pi\gamma} = e \frac{g_{V\pi\gamma}}{4m} \varepsilon_{\mu\nu\lambda\sigma} F^{\mu\nu} V^{\lambda\sigma} \pi. \quad (13)$$

In Fig. 1 we show the Feynman diagrams included in our calculation. From the corresponding matrix elements we extract the photoproduction multipoles. In the first step the isospin amplitudes are calculated from the physical amplitudes:

$$\begin{aligned} M^{3/2} &= M^{\pi^0} - \frac{1}{\sqrt{2}} M^{\pi^+} \\ M^{1/2} &= M^{\pi^0} + \frac{1}{2\sqrt{2}} M^{\pi^+} - \frac{3}{\sqrt{2}} M^{\pi^-} \\ M^0 &= \frac{1}{2\sqrt{2}} (M^{\pi^+} + M^{\pi^-}). \end{aligned} \quad (14)$$

Projecting on total angular momentum  $J$  we obtain the helicity amplitudes

$$H_{\mu,\lambda}^{I,J}(W) = \frac{eM}{8\pi W} 2 \int_0^\pi d\theta \sin \theta M_{\mu,\lambda}^I(W, \theta) d_{\lambda,\mu}^J(\theta). \quad (15)$$

Here  $I$  denotes the isospin channel.  $\mu$  and  $\lambda$  are the final and inital helicities, respectively. The multipole amplitudes are now given by:

$$\begin{aligned} E_{0+}^I(W) &= \frac{\sqrt{2}}{4} (H_{1/2, 1/2}^{I, 1/2} - H_{1/2, -1/2}^{I, 1/2}), \\ M_{1-}^I(W) &= \frac{-\sqrt{2}}{4} (H_{1/2, 1/2}^{I, 1/2} + H_{1/2, -1/2}^{I, 1/2}), \end{aligned}$$

$$\begin{aligned}
E_{1+}^I(W) &= \frac{\sqrt{2}}{8} \left\{ \frac{-1}{\sqrt{3}} \left( H_{1/2, 3/2}^{I, 3/2} - H_{1/2, -3/2}^{I, 3/2} \right) + \left( H_{1/2, 1/2}^{I, 3/2} - H_{1/2, -1/2}^{I, 3/2} \right) \right\}, \\
M_{1+}^I(W) &= \frac{\sqrt{2}}{8} \left\{ \sqrt{3} \left( H_{1/2, 3/2}^{I, 3/2} - H_{1/2, -3/2}^{I, 3/2} \right) + \left( H_{1/2, 1/2}^{I, 3/2} - H_{1/2, -1/2}^{I, 3/2} \right) \right\}, \\
E_{2-}^I(W) &= \frac{\sqrt{2}}{8} \left\{ \sqrt{3} \left( H_{1/2, 3/2}^{I, 3/2} + H_{1/2, -3/2}^{I, 3/2} \right) + \left( H_{1/2, 1/2}^{I, 3/2} + H_{1/2, -1/2}^{I, 3/2} \right) \right\}, \\
M_{2-}^I(W) &= \frac{\sqrt{2}}{8} \left\{ \frac{1}{\sqrt{3}} \left( H_{1/2, 3/2}^{I, 3/2} + H_{1/2, -3/2}^{I, 3/2} \right) - \left( H_{1/2, 1/2}^{I, 3/2} + H_{1/2, -1/2}^{I, 3/2} \right) \right\}. \quad (16)
\end{aligned}$$

### 3 Width parametrizations and cutoff functions

One of the ambiguities introduced in a tree level calculation of the photoproduction amplitudes comes from the inclusion of an energy dependent decay width for the nucleon resonances. As Benmerrouche et al. [11] have shown, unitarity is violated if the denominator of the resonance propagator is simply taken to be  $(q^2 - M_R^2 + iM_R\Gamma(q^2))^{-1}$ . This problem can only be resolved in a K-Matrix approach to meson-nucleon scattering and photoproduction as it has been done for the  $\Delta(1232)$  region and the  $E_{0+}$  multipole in [2, 5].

Since we are mainly interested in the influence of different phenomenological width parametrizations we choose the free decay widths in the pion, eta and two-pion channels calculated using the Lagrangians (1) - (6) times one of the following cutoff factors [3, 12]:

$$F^G(X) = \frac{2}{1 + (X/X_0)^{\alpha_G}}, \quad F^M(X) = \left( \frac{X_0 + x}{X + x} \right)^{\alpha_M}. \quad (17)$$

In the pion and eta decay channels  $X = p^2$ , where  $p$  is the three-momentum of the outgoing meson, for the two-pion case  $X$  is the "free" energy  $X = \sqrt{s} - M - 2m$ . In our work we calculate the total width by summing up the possible partial decay widths using *the same cutoff parameters for all decay channels and nucleon resonances*. This was done in order to avoid introducing too many free parameters. The value  $x = (0.3\text{GeV})^2$  ( $x = 0.3 \text{ GeV}$  for two-pion decays) was fixed in  $\pi N$  scattering [12].

In their model Garcilazo et al. [3] used a simple parametrization of the *total* decay width:

$$\Gamma(s) = \Gamma_0(p/p_0)^{2l+1} \frac{2}{1 + (p/p_0)^{2l+3}}, \quad (18)$$

with  $\Gamma_0$ ,  $p_0$  being the width and the pion momentum at  $s = M_R^2$ . This description might be useful for the  $\Delta(1232)$  channel, but fails to reproduce the  $N(1440)$  and  $N(1535)$  widths since the latter resonances have strong two-pion and eta decay channels, respectively. As an improved description, showing the same threshold and high energy behaviour we used

$$\Gamma(s) = \sum_{i=\pi, \eta, \pi\pi} \Gamma_0^i(s) F^G(X^i) \quad (19)$$

for the width in each possible decay channel.  $\Gamma_0^i(s)$  is the free decay width. The difference of both descriptions for the  $\Delta(1232)$  width is less than 2% around the resonance positions

and of the order of 6% close to threshold. The parametrization (19) therefore allows to treat the decay channels consistently but retains the main features of the one used in [3].

For the cutoff exponents we use  $\alpha_M = l + 1$  for pion and eta decays and  $\alpha_M = 2$  for the two-pion decay <sup>4</sup>. For  $\alpha_G$  Garcilazo et al. used  $(2l + 3)/2$  (18), but we have chosen  $\alpha_G = \alpha_M$  to have the same high energy behavior for both cutoff parametrizations.

The free widths in the pion and eta channels were calculated from the corresponding decay diagram for an "onshell" resonance with mass  $M_R = \sqrt{s}$  [3]. The results for the widths differ by a factor  $(\sqrt{s} + M_R)/2M_R$  from those found from calculating the  $\pi N$  scattering K-Matrix with the given couplings [2]. This factor stems from the projection into the proper partial wave channel that is necessary in the latter case. It can easily be absorbed into the cutoff parametrizations. For the two-pion branch we choose  $\Gamma_{N\pi\pi} = \Gamma_{N\pi\pi}^0 X/X_0$ , with  $\Gamma_{N\pi\pi}^0$  being the free two-pion decay width. This parametrizes the three-particle phase space [4, 13].

Upon including cutoff factors in the width parametrizations consistency would demand a factor  $\sqrt{F^{G,M}}$  in the corresponding meson nucleon Lagrangians, that has been ignored so far in most of the calculations. We will show how the extracted couplings depend on such an extra factor.

Garcilazo et al. have shown that in their calculation it is not possible to reproduce the measured pion photoproduction multipoles without introducing an additional factor  $\Lambda_u^2/(\Lambda_u^2 + p^2)$ ,  $\Lambda_u = 0.3$  GeV for the u-channel resonance diagrams. This is because of the high energy divergence of these contributions which are not reduced by the  $(u - M_R^2)$  denominator in the propagator. Such a cutoff is at this stage purely phenomenological. Furthermore in eta photoproduction a similar cutoff  $(\Lambda_V^2 - m_V^2)/(\Lambda_V^2 - t)$ ,  $\Lambda_V = 1.2$  GeV is used at the  $VNN$  vertex [6]. The dependence of the couplings on these cutoffs will also be considered.

## 4 Photoproduction multipoles

In Table 1 we show the values for masses and widths of the nucleon resonances used in this work. The first set is that of Garcilazo et al., while the second contains the mean values given by the Particle Data Group [7]. Throughout this work the latter values were used. We then extracted the hadronic couplings of the resonances using the formula for the free decay in the corresponding channels. The sign of the couplings was chosen to be positive. The electromagnetic couplings were determined in fits to the photoproduction multipoles using either the resonance masses or the pole positions from [7] <sup>5</sup>.

As a check of our code we compared our results to the figures given by Garcilazo et al. [3]. While we use the same amplitudes as in eqn. (42) of ref. [3], we are unable

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<sup>4</sup>In the case of (pseudo)vector  $\pi, \eta N$  coupling of the spin- $\frac{1}{2}$  resonances one has an additional factor  $(\sqrt{s} + m)^2$  in the free decay width. Therefore we choose  $\alpha_M = l + 2$  in this case to have the same asymptotic behaviour for scalar and vector coupling.

<sup>5</sup>To check the influence of the  $\rho NN$  tensor coupling we performed one fit (# 5) using  $K_V = 6.6$ . With this choice we find a  $\chi^2$  value that is 2% lower than the one given in Table 2. The final parameter estimates show only small deviations from the values found with  $K_V = 3.71$ .

	Garcilazo				PDG 94				
	$M_R$	$\Gamma_\pi$	$\Gamma_\eta$	$\Gamma_{2\pi}$	$M_R$	Pole	$\Gamma_\pi$	$\Gamma_\eta$	$\Gamma_{2\pi}$
$\Delta(1232)$	1.215	0.106	—	—	1.232	1.210	0.120	—	—
$N(1440)$	1.430	0.140	—	—	1.440	1.370	0.228	—	0.122
$N(1520)$	1.505	0.070	—	—	1.520	1.510	0.066	—	0.054
$N(1535)$	1.500	0.060	—	—	1.535	1.500	0.068	0.064	0.018
$\Delta(1620)$	1.620	0.040	—	—	1.620	1.600	0.038	—	0.112
$N(1650)$	—	—	—	—	1.650	1.655	0.105	—	0.019
$\Delta(1700)$	1.700	0.050	—	—	1.700	1.660	0.045	—	0.255

Table 1: Masses and widths used in this work (in GeV). The values of Garcilazo et al. were used to check our calculations (they did not include the  $N(1650)$  and gave no values for the eta and two-pion decay branches). For the PDG case *Pole* gives the average values for the real part of the pole positions.

to reproduce their results. Only after changing the sign of  $K_V$  in (11) we found exact agreement with the calculated amplitudes there. As a further check we calculated the total cross section both from the multipoles and directly using the Feynman matrix elements for the different diagrams. In order to check our  $RN\gamma$  couplings we reproduced the calculation of Pascalutsa and Scholten [14] for Compton scattering on the proton.

Since we want to compare the influence of the different parametrizations in Table 2 we only give  $\chi^2_{norm} = \chi^2/\chi^2_{GG}$  with  $\chi^2_{GG}$  being the value extracted from the corrected model of Garcilazo et al.. Because of the disagreement in the resulting amplitudes mentioned above we first calculated  $\chi^2_{GG}$  within the model given in [3] but using our vector meson couplings from (12). This leads to an increase of  $\chi^2_{GG}$  of 20% as compared to the original calculation of Garcilazo et al.. We then refitted the couplings to obtain a better  $\chi^2_{norm}$  of  $\approx 0.49$  (Fit 1 in Table 2).

In [3] the electromagnetic couplings were extracted from the experimental values for  $A_{1/2,3/2}$  on the resonance points and the  $z_i$  parameters of the spin- $\frac{3}{2}$  resonances were fixed to the values given by Peccei [9],  $z_i = 0.25$ . In order to reproduce the multipole data the masses and widths of the resonances were then adjusted. From comparing the fits 1 and 2 in Table 2 it is clear that the quality of the fit depends strongly on the values of the resonance masses and widths used in [3]. Since the extraction of the  $A_{1/2,3/2}$  parameters depends on a model for pion photoproduction, we have chosen to determine the electromagnetic couplings by fitting the experimental multipoles over the whole energy range instead of the resonance masses and widths, which we take from [7].

The final  $\chi^2$  values are given in Table 2 for the use of both the resonance masses and poles. In general one can see that better fits can be found by using the pole positions instead of the resonance masses. The masses are normally determined in K-Matrix calculations of  $\pi, \eta N$  scattering, whereas the pole positions are taken directly from the

	Resonance values	Width cutoff	$RN\pi$ cutoff	u channel cutoff	$VNN$ cutoff	offshell parameter $z$	$\chi^2_{norm}$ Masses Poles
1	Garcilazo	$F^G$	no	yes	no	$z = -0.25$	0.49 <sup>1</sup>
2	PDG 94	$F^G$	no	yes	yes	$z = -0.25$	1.95 1.47
3	PDG 94	$F^G$	no	yes	yes	Davidson <sup>2</sup>	6.37 6.01
4	PDG 94	$F^G$	no	yes	yes	Davidson <sup>3</sup>	2.88 2.13
5	PDG 94	$F^G$	no	yes	yes	Fit	1.47 0.51
6	PDG 94	$F^M$	no	yes	yes	Fit	1.55 0.66
7	PDG 94	$F^G$	yes	yes	yes	Fit	2.66 1.93
8	PDG 94	$F^M$	yes	yes	yes	Fit	2.33 1.33
9	PDG 94	$F^G$	no	yes	no	Fit	1.51 0.47
10	PDG 94	$F^G$	no	no	yes	Fit	1.71 1.73

Table 2:  $\chi^2_{norm}$  of our fits as described in the text, using the different width parametrizations and cutoff factors. The first values give the  $\chi^2_{norm}$  for the calculations with the resonance mass, the second are for the pole positions. (<sup>1</sup> : Refitted couplings within the model of Garcilazo et al.. The first set of Table 1 was used, but the vector meson couplings from (12), <sup>2</sup> : The values from [2] were used for all spin- $\frac{3}{2}$  resonances, <sup>3</sup> : The offshell parameters of the  $N(1520)$  and  $\Delta(1700)$  were allowed to vary)

corresponding experimental multipoles.

In total 10 fits were performed for different combinations of resonance data and cutoff factors. This allows to investigate the sensitivity of the extracted resonance parameters on the various parametrizations given in the last section. Table 2 shows that the quality of all fits with the exception of no. 3 is comparable, with fits 5, 6 and 9 being the best. The equal quality of fits 5 and 6 shows that the shape of the width cutoffs (17) is not essential.

In one case (fit 3 in Table 2) the offshell parameters  $z_i$  of the spin- $\frac{3}{2}$  resonances were fixed to the values given by [2],  $z_\pi = -0.24, z_1 = 2.39, z_2 = -0.53$ . With this choice the overall  $\chi^2$  increases by a factor of about 9 (see Fig. 2). As is shown in Fig. 3 for the  $E_{0+}^p$  multipole this is due to the offshell contribution of the  $D_{13}$   $N(1520)$  resonance which depends strongly on the choice of the  $z_i$  parameters. To investigate this dependence in more detail we have performed one fit (4) where only the  $\Delta(1232)$  offshell parameters were taken from [2] and the values for the  $N(1520)$  and  $\Delta(1700)$  were allowed to vary. The resulting lower  $\chi^2$  shows that the multipole data are highly sensitive to the  $N(1520)$  offshell contributions. Especially the  $E_{0+}^p$  multipole imposes strict limits on the  $z_i$  parameters.

When the  $\Delta(1232)$  offshell parameters are also fitted to the data,  $\chi^2$  decreases for both choices of the cutoff factor  $F^{G,M}$ . The final values for the  $z_i$ 's differ strongly from those given by Davidson et al.. This is probably due to the missing rescattering in our calculation. Both offshell contributions and rescattering effects are most effective in channels



that are not strongly dominated by one resonance (eg. in  $E_{1+}^{3/2}$ ). So during the fitting the  $z_i$  parameters adjust to compensate for the lack of rescattering even though both effects result from totally different physical mechanisms.

As shown in Table 2 two fits (7 and 8) were made using a cutoff factor  $\sqrt{F^{G,M}}$  at the  $RN\pi$  vertices. Both of these cutoffs show the same high energy dependence (see Eq. (17)). Below threshold they are larger than 1 and thus enhance the resonance contributions (about a factor  $\sqrt{2}$  for  $F^G$  and  $2 - 30$  for  $F^M$  depending on the mass and the angular momentum of the decay pion). For higher energies however, they both lead to about the same reduction of the corresponding amplitudes. As Table 3 shows, which compares one of those fits (7) with the fit no. 5, the extracted electromagnetic couplings are drastically different in both cases. This was to be expected since both lead to a rather large  $\chi^2$  value and therefore do not determine the resonance parameters very accurately. When using the resonance pole positions the final couplings for the fits 5 and 7 show a somewhat better agreement (Table 4).

When using the resonance pole positions part of the rescattering effects are taken into account by the mass shift. This is the main reason for the lower  $\chi^2$  values found in these fits. Already the shift of the  $\Delta(1232)$  resonance can explain a large part of this effect. Since the corresponding  $M_{1+}^{3/2}$  multipole is dominant in the pion photoproduction, the value of the  $\Delta(1232)$  mass enters crucially into the calculations. As can be seen from Table 1 this value is nearly the same in our calculation and in the work of Garcilazo et al.. So it was to be expected that their model leads to about the same  $\chi^2$  values as our best fits using the resonance pole positions.

A comparison of the final parameter estimates in Table 4 shows that the  $\Delta(1323)$  couplings are rather insensitive to the different prescriptions used in the fits. The offshell parameters of Fits 5 and 9 are comparable while the result of Fit 7 shows large deviations. Since there is no interference with other resonance contributions in the  $\Delta(1323)$  energy region these differences are directly related to the different cutoffs used in the fits. For the  $N(1520)$  and  $\Delta(1700)$  resonances this is not true. In this energy region we have resonances in most of the channels where there are also large offshell contributions. Therefore the  $z_i$  parameters are not as uniquely determined as in the  $\Delta(1323)$  case. By comparing the Fits 5 and 7 with 9 one sees that different combinations of resonance couplings of the spin- $\frac{1}{2}$  resonances and offshell parameters can lead to about the same  $\chi^2$ . Especially for resonances with weak electromagnetic couplings ( $\Delta(1620)$  and  $N(1650)$ ) this leads to drastically different parameters.

In order to check the dependence of the couplings on the use of the u-channel and vector meson cutoffs we tried to fit the data without either of these. The corresponding  $\chi^2$  values are given in the last two lines of Table 2. Comparing fits 5 and 9 one can see that these are not very sensitive to a cutoff at the VNN vertex. Only for the pole positions we find differences to the fits done with this cutoff. The lower  $\chi^2$  value results from cancellations of different contributions at higher energies. These can only take place if the vector meson amplitude is not reduced by a cutoff. In Tables 3 and 4 we give the extracted couplings for fit 9.

For a u-channel cutoff the situation is different. Garcilazo et al. showed that they

Spin - $\frac{1}{2}$			Spin - $\frac{3}{2}$						Fit
	$g_p$	$g_n$		$g_1$	$g_2$	$z_\pi$	$z_1$	$z_2$	
$N(1440)$	-0.448	0.115	$\Delta(1232)$	4.774	8.467	-0.308	-1.141	0.666	5
$N(1535)$	0.797	-0.413	$N(1520)$	3.295	3.020	-2.492	-0.141	-0.103	
$\Delta(1620)$	-0.348	—		-1.170	1.110				
$N(1650)$	0.092	0.185	$\Delta(1700)$	1.417	2.885	0.098	-0.460	0.801	
$N(1440)$	-0.404	0.313	$\Delta(1232)$	5.478	7.611	-0.594	0.050	1.499	7
$N(1535)$	0.649	-0.544	$N(1520)$	4.914	5.061	-0.617	0.126	0.889	
$\Delta(1620)$	-0.131	—		-1.387	0.677				
$N(1650)$	0.154	-0.023	$\Delta(1700)$	1.038	2.542	0.170	0.131	-2.292	
$N(1440)$	-0.442	0.101	$\Delta(1232)$	4.784	5.179	-2.402	-0.300	-0.136	9
$N(1535)$	0.692	-0.209	$N(1520)$	3.095	3.003	-1.817	-0.092	-0.024	
$\Delta(1620)$	-0.109	—		-1.839	0.018				
$N(1650)$	0.089	0.186	$\Delta(1700)$	1.625	3.242	-1.405	-0.266	0.042	

Table 3: Final parameter estimates for fits 5, 7 and 9 of Table 2 using the resonance masses. Left side: spin- $\frac{1}{2}$ , right side: spin- $\frac{3}{2}$  resonances (For the  $N(1520)$  proton couplings are given in the first, neutron couplings in the second line).

Spin - $\frac{1}{2}$			Spin - $\frac{3}{2}$						Fit
	$g_p$	$g_n$		$g_1$	$g_2$	$z_\pi$	$z_1$	$z_2$	
$N(1440)$	-0.400	0.110	$\Delta(1232)$	5.416	6.612	1.400	-0.293	-0.394	5
$N(1535)$	0.623	-0.583	$N(1520)$	3.449	3.003	-2.418	-0.158	-0.160	
$\Delta(1620)$	-0.144	—		-0.307	1.862				
$N(1650)$	0.205	0.200	$\Delta(1700)$	1.895	3.921	-0.606	-1.120	0.124	
$N(1440)$	-0.403	0.137	$\Delta(1232)$	5.498	6.220	-0.474	-0.196	1.212	7
$N(1535)$	0.733	-0.516	$N(1520)$	4.899	5.074	-2.326	-0.047	0.148	
$\Delta(1620)$	-0.003	—		-0.378	0.719				
$N(1650)$	0.279	0.147	$\Delta(1700)$	1.335	2.694	-0.398	-1.032	0.204	
$N(1440)$	-0.401	0.186	$\Delta(1232)$	5.099	5.836	0.949	-0.310	-0.483	9
$N(1535)$	0.627	-0.474	$N(1520)$	3.004	3.047	-0.123	1.390	0.267	
$\Delta(1620)$	-0.005	—		-0.068	1.265				
$N(1650)$	0.045	0.011	$\Delta(1700)$	1.516	2.176	0.016	0.978	-2.297	

Table 4: Same as Table 3, but for the resonance pole positions.

needed this cutoff to suppress the contribution to the multipoles from the crossed resonance diagrams. A detailed analysis of their results shows that the  $\Delta(1232)$  accounts for most of the divergence they find. Our fit indicates that this conclusion strongly depends on the choice of the offshell parameters  $z_i$  used. If these are allowed to vary, we find parameter sets that lead to large cancellations between the offshell contributions of different resonances. However, one can see from the fit using the resonance poles that the  $\chi^2$  is not reduced in this case as it was for the other fits. This shows that the additional degrees of freedom from the  $z_i$  parameters are used to mock up the effect of an u-channel cutoff without leading to an improvement of other features. We thus conclude, in agreement with [3], that without such an u-channel cutoff no satisfactory description of the multipole data can be found.

In addition to the  $E_{0+}^p$  multipole we show in figs. 4 - 5 the result for all multipoles calculated using the parameters of Fit 5 using the resonance pole positions. In general we have good agreement with the experimental data in all channels.

Clearly visible is the strong peak in the  $E_{0+}^p$  multipole because of the reduced  $N(1535)$  mass. In the  $M_{1\pm}^p$  channels the  $N(1520)$  offshell contributions are most prominent. Especially for the imaginary part this stands in contrast to the experimental values. For the  $z_i$  parameters used by Garcilazo et al. only the  $M_{1+}^p$  multipole shows this behaviour. But no choice of the offshell parameters could be found that reduces the effect in both multipole channels. It would be interesting to see the effect of rescattering on this results since on the Lagrangian level only a drastic cutoff of the crossed resonance diagrams could remove this contributions.

The missing rescattering is obvious in the  $E_{1+}^{3/2}$  channel. Here it is well known that only a complete K-matrix calculation can reproduce the data [11]. The same seems to be true for the  $M_{2-}^n$  multipole where one has a similar situation. An otherwise strong spin- $\frac{3}{2}$  resonance ( $N(1520)$ ) with a weak coupling into this particular channel. Therefore this multipole would also be very sensitive to rescattering effects.

## 5 Conclusions

In this work we have determined the electromagnetic couplings of nucleon resonances by fitting the pion photoproduction multipoles for various choices of resonance values and cutoffs. The importance of pion photoproduction as the main reaction to extract these couplings [7] shows how necessary a detailed understanding of all model dependencies is. It was found that different values of the electromagnetic couplings in combination with different choices for the width parametrizations lead to fits of equal quality. Only for the resonances with strong electromagnetic decay channels the final coupling parameters are comparable. For the other resonances ( $\Delta(1620)$  and  $N(1650)$ ) the couplings are not well determined by this calculation.

For the use of the resonance masses all extracted couplings are very sensitive to the cutoffs used in the calculation. We find reasonably stable electromagnetic couplings only for the fits with the resonance pole positions. The values that we then find are in agreement with those given by other calculations [7].

The other important point in our view is the investigation of the offshell contributions of the spin- $\frac{3}{2}$  resonances. Besides the resonance mass values the offshell parameters  $z_i$  have the biggest influence on the quality of the fits. Our calculation shows that this is true for both the  $\Delta(1232)$  and the  $N(1520)$  resonance, with the latter showing major contributions to the  $E_{0+}^p$  multipole. Since the interpretation of the  $z_i$  parameters from a fieldtheoretical point of view is still unclear and since there is no prediction for the exact values there is clearly a need for their determination from experimental data. Here also calculations of other reactions are necessary because otherwise the large number of free parameters does not allow to give strict limits on the extracted couplings.

Furthermore we confirm the finding of Garcilazo et al. that the multipole data can only be reproduced by using an u-channel cutoff for the resonance contributions. Otherwise these divergent amplitudes dominate the calculated multipoles for higher energies. For the different cutoffs used in the width parametrizations one can say that the quality of the fits does not depend on their exact form. Any cutoff that leads to a decrease of the width beyond the resonance position would give similar  $\chi^2$  values.

As we have already discussed, the better fits using the resonance pole positions show that the rescattering needs to be included in our calculation. This would also limit the number of free parameters in the photoproduction since all hadronic parameters would then be uniquely defined by the other reaction channels.

From the calculation of Sauermann et al. [5] the influence of the rescattering effects in the  $E_{0+}^p$  channel can be estimated. Since for most of our fits the offshell  $N(1520)$  contributions are of the same order of magnitude (comp. Fig. 3) one can compare rescattering and offshell effects without depending on the exact choice of coupling parameters. In doing so one has to keep in mind that in our calculation part of the rescattering is already taken care of by the decay width of the nucleon resonances. This corresponds to 'direct' rescattering going through the same resonance in a K-matrix calculation. So only the 'indirect' rescattering (through background and other resonances) is missing in our calculation. From [5, 15] and our calculations we find that this 'indirect' part and the offshell  $N(1520)$  contributions are of equal importance. Therefore it is not possible to neglect either of these effects without introducing large uncertainties in the extracted resonance electromagnetic couplings.

Our results clearly show the need for a complete K-matrix calculation including all resonances and multipole channels. Due to the offshell  $N(1520)$  contribution the multipole decomposition does not allow to disentangle the different nucleon resonances. Especially the extracted  $N(1535)$  and  $N(1650)$  resonance parameters using the  $S_{11}$   $\pi N$  channel and the  $E_{0+}^p$  photoproduction multipole [5] are not independent of the  $N(1520)$  couplings. Also the high sensitivity of our fits on the resonance masses indicate that a simultaneous determination of both masses and hadronic as well as electromagnetic couplings is needed.

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# Figures

Figure 1: Feynman diagrams for pion photoproduction. From left to right: direct graph, exchange graph, pion pole or vector meson graph, seagull graph.

Figure 2: Real and imaginary part of the  $E_{0+}^p$  photoproduction multipole in millifermi units for different fits. For comparison we show the result of Garcilazo et al.. Solid lines: results of [3], dashed: using parameters of Fit 3, dashed-dotted: Fit 7. The data are taken from [16].

Figure 3: Same as Fig. 2, but for the  $N(1520)$  s- and u-channel contributions only.

Figure 4: Electromagnetic multipoles for the isospin- $\frac{1}{2}$  channel and proton target in millifermi calculated using the couplings of Fit 5 with the resonance pole positions. The solid (dashed) lines are the real (imaginary) parts. Solid (open) squares are the experimental data of [16]. The marks on the energy axis indicate that for certain multipoles no single-energy-data was available in this energy range.

Figure 5: Same as Fig. 4 but for the neutron target.

Figure 6: Same as Fig. 4 but for the isospin- $\frac{3}{2}$  channel.

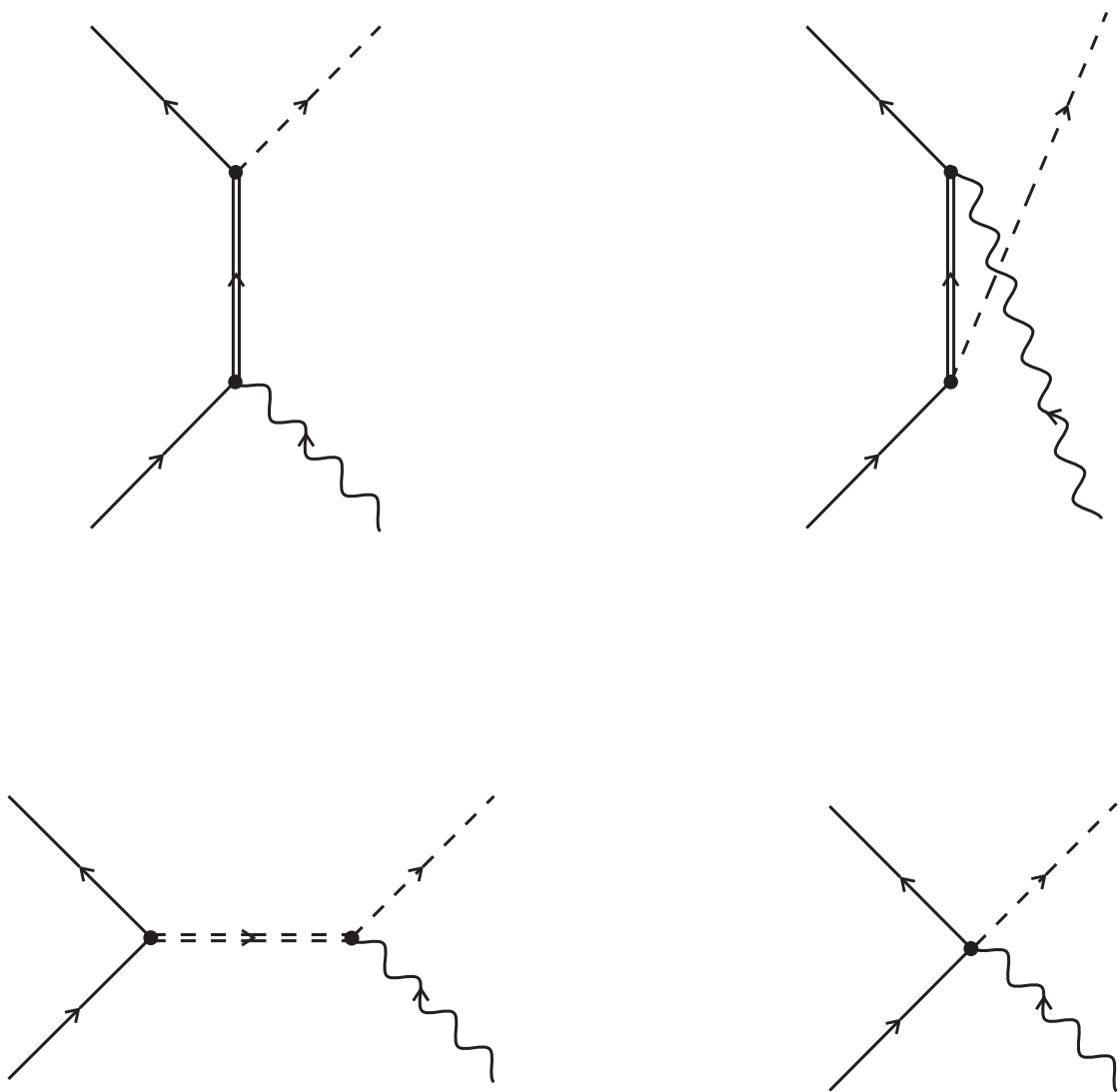


Figure 1: Feynman diagrams for pion photoproduction. From left to right: direct graph, exchange graph, pion pole or vector meson graph, seagull graph.

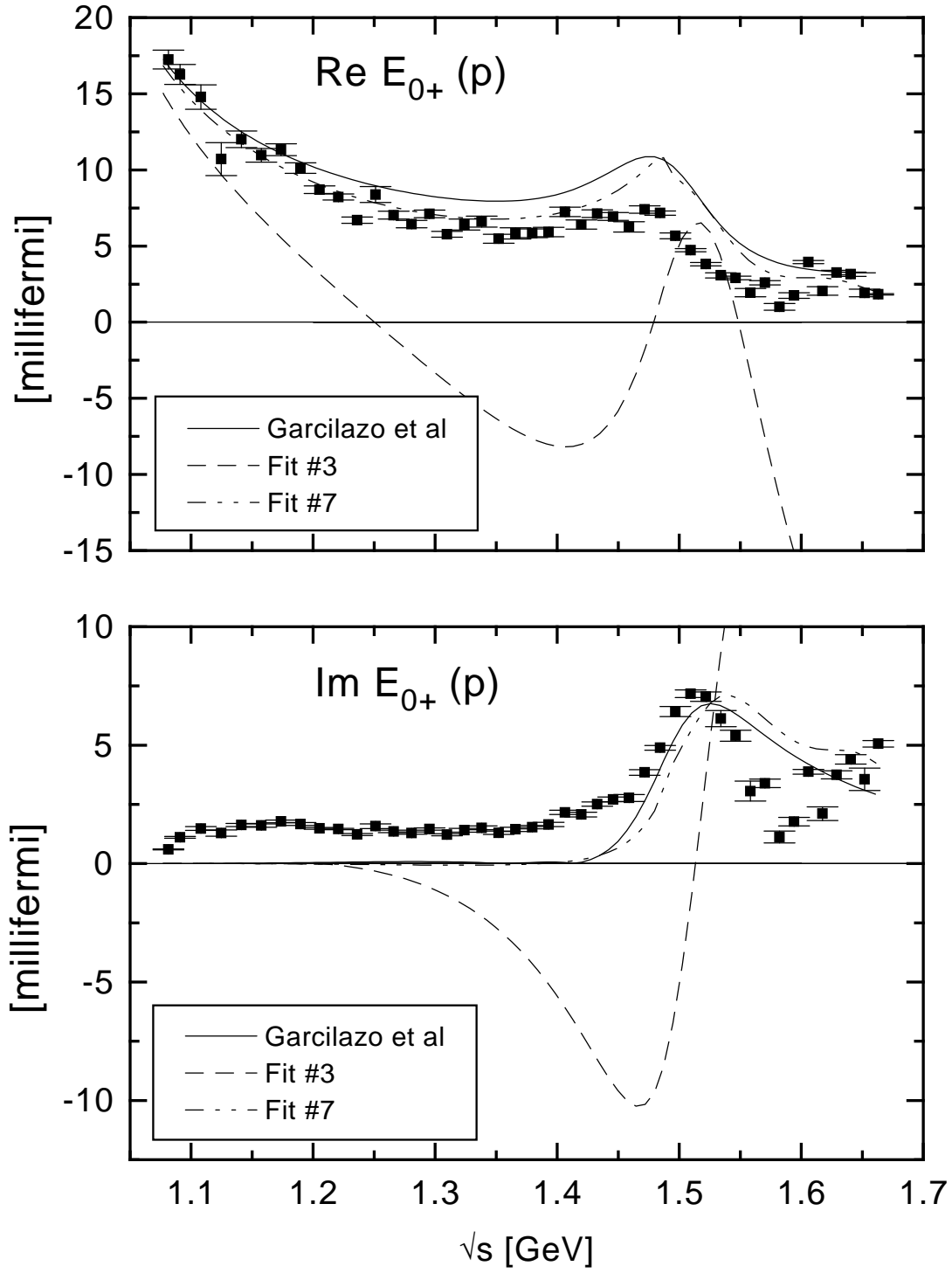


Figure 2:  $E_{0+}^p$  pion photoproduction multipole, all contributions.



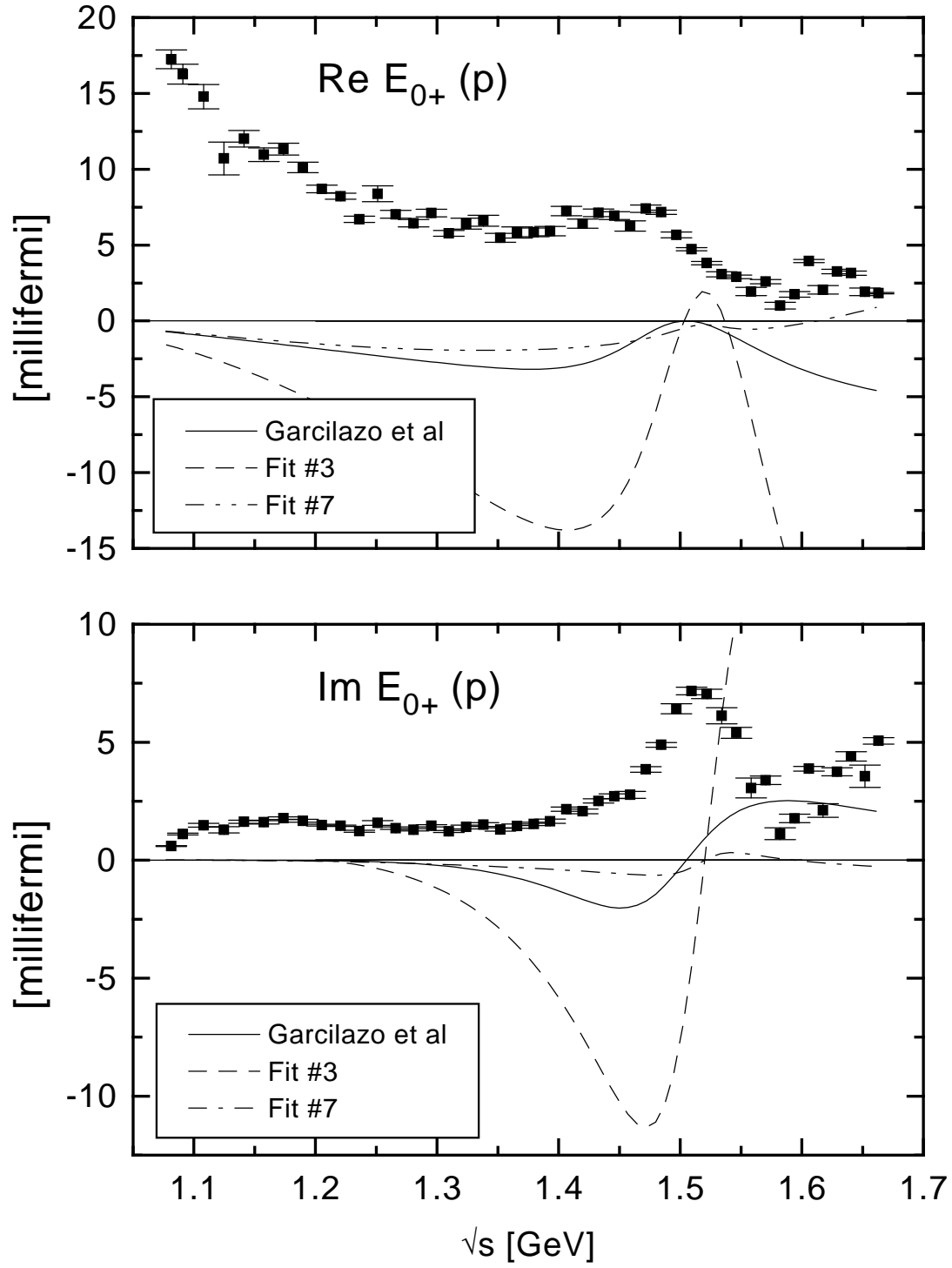
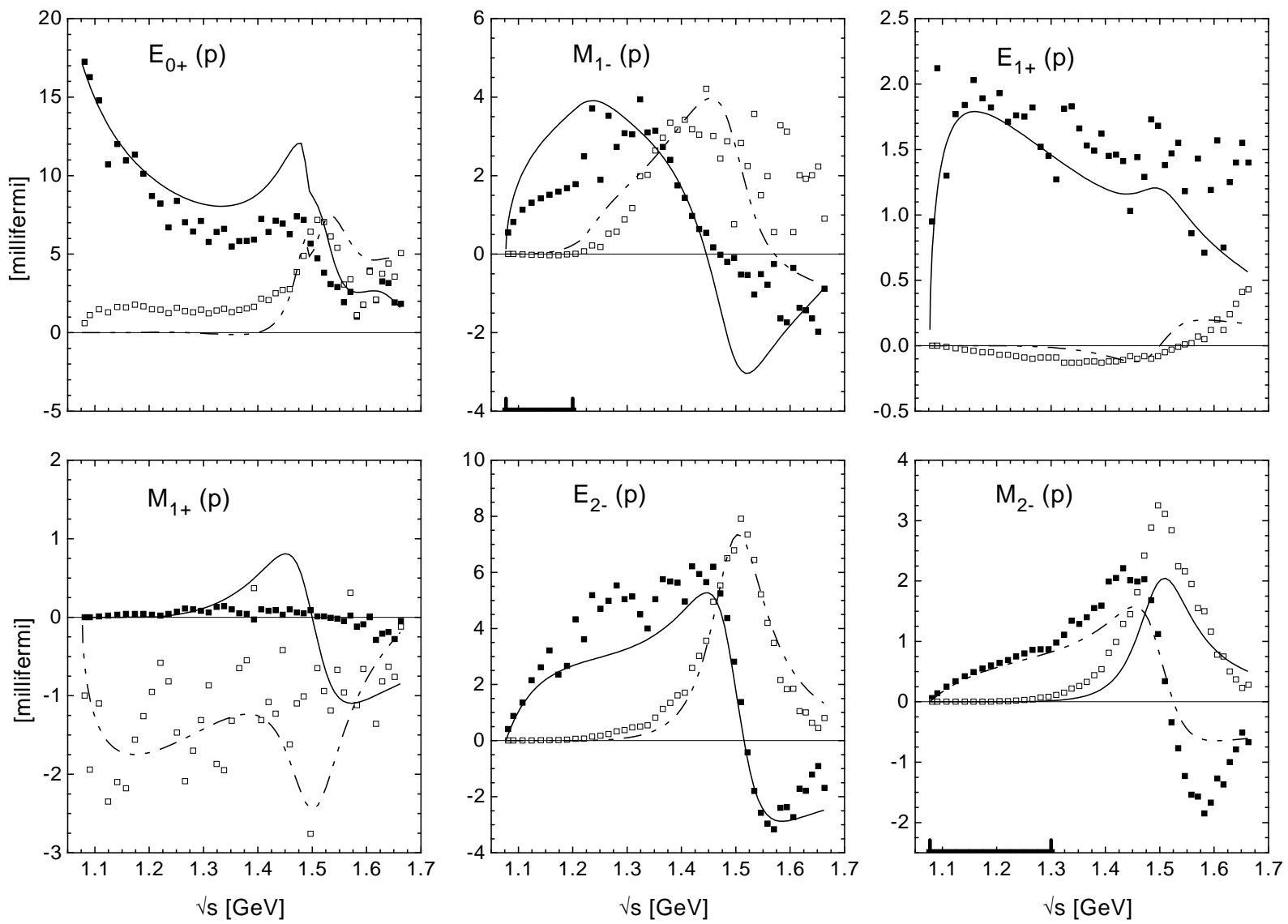


Figure 3:  $E_{0+}^p$  pion photoproduction multipole,  $N(1520)$  s- and u-channel contributions only.

Figure 4: Electromagnetic multipoles for the isospin- $\frac{1}{2}$  channel and proton target.



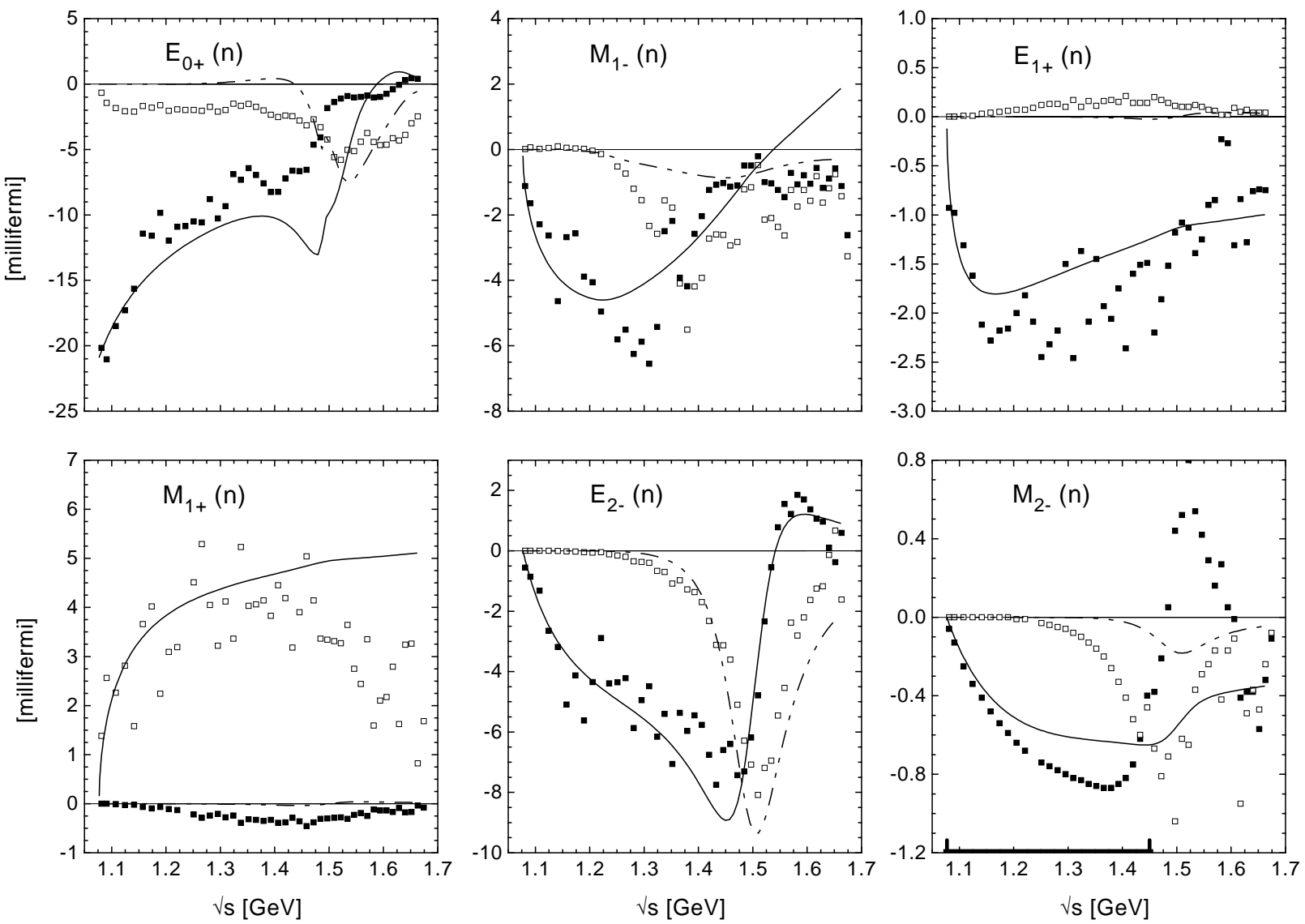


Figure 5: Electromagnetic multipoles for the isospin- $\frac{1}{2}$  channel and neutron target.

Figure 6: Electromagnetic multipoles for the isospin- $\frac{3}{2}$  channel.

